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15MAT11

First Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics - I

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions.***

1 a. Find the n^{th} derivative of $\cos x \cos 3x \cos 5x$. **(06 Marks)**
b. Obtain the Pedal equation of the curve $r = 2(1 + \cos\theta)$. **(05 Marks)**
c. Find the radius of curvature of the curve $x = a\log(\sec t + \tan t)$, $y = a\sec t$. **(05 Marks)**

2 a. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(06 Marks)**
b. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect each other Orthogonally. **(05 Marks)**
c. Show that for the curve $r(1 - \cos\theta) = 2a$, ρ^2 varies as r^3. **(05 Marks)**

3 a. Obtain the Maclaurin's expansion of $\log(1 + e^x)$ as far as the fourth degree terms. **(06 Marks)**
b. Evaluate : $\underset{x\to 0}{\mathrm{Lt}}\left[\frac{1}{x^2} - \frac{1}{\sin^2 x}\right]$. **(05 Marks)**
c. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(05 Marks)**

4 a. Evaluate : $\underset{x\to 0}{\mathrm{Lt}}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$. **(06 Marks)**
b. If $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. **(05 Marks)**
c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at (1, -1, 0). **(05 Marks)**

5 a. A particle moves along the curve, $x = 1 - t^3$, $y = 1 + t^2$ and $z = 2t - 5$.
i) Determine its velocity and acceleration.
ii) Find the components of velocity and acceleration at $t = 1$ in the direction $2i + j + 2k$. **(06 Marks)**
b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2). **(05 Marks)**
c. Prove that Curl (grade ϕ) = $\vec{0}$. **(05 Marks)**

6 a. Find the directional derivatives of $\phi = x^2yz + 4xz^2$ at (1, -2, -1) along $2i - j - 2k$. **(06 Marks)**
b. Show that $\vec{F} = 2xyz^2 i + (x^2z^2 + z\cos(yz))j + (2x^2yz + y\cos(yz))k$ is a potential field and hence find its scalar potential. **(05 Marks)**
c. Prove that div(Curl $\vec{A}$) = 0. **(05 Marks)**

7 a. Obtain the reduction formula for $\int \sin^n x dx$. **(06 Marks)**

b. Show that the family of parabolas $y^2 = 4a(x + a)$ is self Orthogonal. **(05 Marks)**

c. Solve $y\, e^{xy}\, dx + (x\, e^{xy} + 2y)dy = 0$. **(05 Marks)**

8 a. Obtain the reduction formula for $\int \sin^m x \cos^n x\, dx$. **(06 Marks)**

b. Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 x$. **(05 Marks)**

c. A body in air at 25°C cools from 100°C to 75°C in 1 minute. Find the temperature of the body at the end of 3 minutes. **(05 Marks)**

9 a. Find the rank of the matrix by elementary row transformation.

$$A = \begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$ **(06 Marks)**

b. Apply Gauss – Jordan method to solve the system of equations : $2x + 5y + 7z = 52$; $2x + y - z = 0$; $x + y + z = 9$. **(05 Marks)**

c. Show that the transformation : $y_1 = 2x_1 - x_2 - x_3$, $y_2 = -4x_1 + 5x_2 + 3x_3$, $y_3 = x_1 - x_2 - x_3$ is regular and find the inverse transformation. **(05 Marks)**

10 a. Solve $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$ by Gauss – Seidel method. **(06 Marks)**

b. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(05 Marks)**

c. Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1 x_3$ to Canonical form. **(05 Marks)**

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